A Structural Approach for Estimating the Value of Public Goods in a Cause-related Market

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Abstract

People often pay a premium for products linked to charity due to the belief that part or all the revenue generated from such purchases is a public good. However, accurately quantifying the value of public goods is challenging due to identification issues. This study aims to address this challenge by leveraging two sets of bidding data from eBay on charity and non-charity auctions of identical products. I employ an nonparametric approach to identify bidders' public goods values within an independent private values paradigm, providing a unique and valuable insight into the motivations behind charitable giving through auction data. Specifically, by using the variation in donation percentages in the charity auction set, this study develops structural altruistic estimators to uncover each bidder's latent private value, which is a combination of the value of the auction item and the linked public good. In the non-charity auction set, the latent private value of each bidder is identified based on the auction item alone. Then the difference between the latent private values of the two sets is used to determine the value placed on charity-linked products, assuming that the two sets have the same underlying distribution of private values on the auction item. The results show that on average, bidders bid 6% more in charity auctions compared to non-charity auctions, driven primarily by the warm glow of giving, the joy from the act of giving.

Keywords: Charity, Second Price Auction, Nonparametric Methods

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1 Introduction

The cause-related market, encompassing the size, funding flow, corporate involvement, and consumer motivations, is often underappreciated. The Product Red campaign serves as a compelling example, where purchasing Red-branded items from companies such as Gap Inc., Apple Inc., Dell Inc., and Starbucks Corp., consumers can support non-profit organizations such as the Global Fund to fight AIDS, Tuberculosis, Malaria, and COVID-19. Apple Inc., one of the most well-known Red partners, has raised over $270 million in the fight against global pandemics, with over $21 million donated to the Global Fund's COVID Response Fund. Since its inception in 2006, Red and its partners have raised more than $700 million for the Global Fund's efforts to combat AIDS.[1]

Amidst this growing landscape, an increasing number of corporations are recognizing their social responsibilities and actively participating in the cause-related market. Recent additions like Chase and Wells Fargo have embraced the cause, supporting endeavors that provide urgent and equitable access to COVID relief in impoverished countries. Additionally, 86% of Americans express their likelihood to purchase products from purpose-driven companies, with 72% saying it is more important than ever for the companies they buy from to reflect their values during the COVID-19 pandemic.[2] However, despite the obvious and growing size, significance, and strong consumer preference for the cause-related market, there remains an incomplete understanding of the motivations behind consumers' purchases of these products

One possible explanation is that people value public goods. The ensuing literature is too large to cite completely here. A sampling of papers that conclude people value public goods with a quasi-linear preference for consumers includes but is not limited to Morgan (2000),Fehr and Gächter (2000), and Morgan and Sefton (2000). Researchers have used log-linear preference in studies as well, such as Ottoni-Wilhelm et al. (2017), finding that people

[1]The information comes from https://summer.red.org/give/

[2]The information comes from 2020 Porter Novelli/ Cone Purpose Biometrics Study.

give for reasons of both pure altruism and warm glow. This finding leads to a parametric specification of the bidders' values of public goods. Despite this extensive research, there has been limited investigation into the distribution of public good values. In contrast, this paper aims to address this gap by exploring the possibility that people value public goods without assuming a specific parametric specification of their values. Additionally, we propose structural altruistic estimators that can be used to estimate the distribution of public good values.

In this study, we build structural models for second-price auctions, considering both scenarios with and without charitable donations within an independent private value paradigm. By using the variation in donation percentages in ovserved bids, we structurally estimate the parameters of altruism, including pure altruism and warm glow. These altruism parameters are then used to derive the pseudo-private value in each auction through bidding functions, both for charity and non-charity auction sets. For charity auctions, we estimate the joint value of the auction item and public good, accounting for both the consumption of the auction item and the public good value derived from the donation to charity. Conversely, for non-charity auctions, we estimate the bidders' private value on the auction items only.

Under the assumption that the underlying distribution of private values for auction items is the same for both charity and non-charity auctions, we calculate the value of public goods by comparing the latent private values across these auction types. On empirical grounds, this assumption requires the same bidders participate in both auctions with and without charity. To empirically validate this assumption, we prove it using a subsample of our data that has the same bidders active in both matched charity and non-charity auctions. Consequently, the difference between the joint valuation and the valuation for auction item can then be considered as the valuation of public goods. Moreover, using our data from eBay that comprises auctions with and without charitable donations of otherwise identical products, we test how well our approach works in the field.

The approach presented above builds upon prior research in the field. The most relevant studies include Lu and Perrigne (2008),Engers and McManus (2007) and Fioretti (2020). In particular, we closely follow the methodology of Engers and McManus (2007) in developing our charitable auction model within the context of a second-price auction. Engers and McManus (2007) work out the bidding strategies in pure public good contexts, where all auction revenue goes to charity. Our research extends this model to a cause-related market, in which a portion or all proceeds go to charity and bidders not only value the private consumption of the auction item, but also derive extra benefits from the charitable donation. To determine the joint value of the auction item and charity, we aim to identify the altruistic parameters in a sealed second-price auction with a charity component. For this purpose, we utilize the identification methodology from Lu and Perrigne (2008), which also has been employed by Fioretti (2020). Lu and Perrigne (2008) use this method to estimate bidders' risk preferences by comparing bidders' behavior in ascending and sealed first-price auctions. Fioretti (2020) investigates the objectives of firms in a cause-related market using this method. In our paper, we use this approach to quantify the altruistic parameters and determine the distribution of values of public goods in a cause-related market.

Our methodology provides a comprehensive framework for evaluating the private values and motivations that drive the charity premium within the independent private values framework. This approach has the potential to enhance the effectiveness of tax policy design by taking into account the role of altruistic behavior in determining the sensitivity of charitable donations to price changes, which is a critical element in formulating an optimal tax policy. Moreover, it enables firms to better understand the impact of offering cause-related products on their profits and charity fundraising, enabling them to design more effective corporate social responsibility strategies. This is made possible by our structural approach, which allows for simulations with varying donation percentages, and its nonparametric nature, which avoids potential identification issues, such as risk preferences. Importantly, this methodology can be readily adapted by other researchers to investigate a broad spectrum of questions in various social sciences, extending its applicability beyond the scope of this study.

In addition to the methodology contribution to the literature on tax policy and social responsibility, our study makes a contribution to the literature on charity auctions as well. Although previous research has shown that bidders take charity into consideration in charity auctions, few studies have quantified the value of public goods in these auctions.[3] Elfenbein and McManus (2010) found that, on average, bidders bid 6% more for charity-linked auctions than non-charity auctions. In contrast, our study establishes a way to explore the charity premium nonparametrically, along with the marginal value of charity and the distribution of the value of charity revenue as a public good in charity auction context. By adopting this nonparametric framework, we are able to capture a more nuanced understanding of implications of the charity premium, providing valuable insights into the underlying mechanisms at play. Most importantly, these results are useful in social welfare improvement. If governments encourage firms to support the charity most in need through policy incentives, social welfare can be improved saliently.

Using this approach along with the data from eBay, we find that on eBay, bidders bid, on average, 6% more in charity auctions than in non-charity auctions, which is consistent with the reduced-form estimation. bidders' higher payments for charity-linked products are mainly motivated by warm glow. Our results show that the magnitude of warm glow is 9 cents for every dollar donated to charity, while pure altruism is positive, although close to zero for every one-dollar donation. Furthermore, we find that the private values from charity and non-charity auction sets are significant. To the best of our knowledge, this is the first study to investigate the distribution of charitable contributions in an auction context using field data.

The rest of this paper proceeds as follows. The following section discusses prior literature including literature on charity auctions and the structural analysis of auction data. Section

[3]See,Ottoni-Wilhelm et al. (2017),Andreoni (1989),Leszczyc and Rothkopf (2010),Fioretti (2020),Casadesus-Masanell et al. (2009),J.Hiscox et al. (2015),Elfenbein and McManus (2010), and Leszcayc et al. (2013).

3 presents the theory about second-price auctions with and without charity, as well as the methods for identifying altruism parameters and the distribution of public good values. In this section, we build our second-price model for cause-related market based on Engers and McManus (2007), and develop our identification following closely Lu and Perrigne (2008) and Fioretti (2020). In Section 4 we describe the data used in this study. Section 5 includes Monte Carlo simulations to assess the reliability of our identification methods for altruistic parameters and the distribution of bids obtained from winning bids only. In Section 6, we address potential concerns about our methodologies. Our empirical analysis and robustness tests are presented in Section 7. In the final section, the paper draws its conclusions.

2 Literature Review

2.1 Charity auctions

Both theoretic and empirical research indicate that people are willing to pay more in charity auctions than in non-charity auctions (Engers and McManus (2007), Elfenbein and McManus (2010), Leszczyc and Rothkopf (2010), Haruvy and Leszcayc (2009), Leszcayc et al. (2013), and Hiscox et al. (2011)). For example, Elfenbein and McManus (2010) analyze data from eBay on charity and non-charity auctions of otherwise identical products and find that prices in charity auctions are 6 percent higher, on average, than them in non-charity auctions. Haruvy and Leszcayc (2009) find that bidders are willing to pay a premium in charity auctions. Leszcayc et al. (2013) identify charitable and non-charitable bidders and find that charitable bidders are willing to pay a significant premium in charity auctions and bid more persistently. Charitable bidders are more likely to bid in future charity auctions even after losing one. Hiscox et al. (2011) find that bidders bid, on average, 23% higher for coffee labeled Fair Trade using an experiment conducted on eBay.

However, most theoretical papers in charity auctions focus on the comparison of different charity auction formats and empirical papers try to test the predictions of theoretical papers. Some research has tended to support that all-pay auction format will generate higher revenue than winner-pay auction format (Engers and McManus (2007), Schram and Onderstal (2009), Carpenter et al. (2010), Engelbrecht-Wiggans (1994), Bos (2020), Bos et al. (2021)). Engers and McManus (2007) compare equilibrium bidding and revenue in first-price, second-price, and all-pay charity auctions. They find that first-price auctions generate less revenue than both second-price and all-pay auctions. Between second-price and all-pay auctions, all-pay auction has higher bidding revenue if the number of bidders is high enough. Also, they find all three auction formats would generate the same bidding revenue if and only if the auctioneer can set a reserve price and fees plus threaten to cancel the auction. Schram and Onderstal (2009) compare first-price winner-pay auctions, first-price all-pay auctions, and lotteries. They find the all-pay format raises relatively higher revenue than the other two mechanisms. Carpenter et al. (2010) examine comprehensively all 15 existing charity auction formats. Bos (2020) finds that the all-pay auction is better at raising money for charity than the first-price auction with asymmetric bidders under some incomplete information. Exceptions exist, for example, Bos (2016) demonstrates theoretically that first-price and second-price winner-pay auctions have a better revenue performance than first-price and second-price all-pay auctions when bidders are sufficiently asymmetric under complete information.

Within the winner-pay auction mechanism, Carpenter et al. (2011) show silent charity auctions outperform the English auction when charities can encourage jumping and discourage sniping.[4][5] Engelbrecht-Wiggans (1994), Bos (2016), and Engers and McManus (2007) find that second-price sealed bid auctions do better than first-price auctions.

Between voluntary-pay and winner-pay auctions in charity setting, Haruvy and Pop-

[4]More studies within the winner-pay auction context (Bos, 2016; Carpenter et al., 2011; Engelbrecht-Wiggans, 1994; Engers and McManus, 2007).

[5]In silent auctions, jumping is the practice of bidders increasing their bids significantly beyond the minimum increment. Sniping, on the other hand, is a strategic tactic where bidders wait until the final moments of the auction to place their initial bid.

kowski Leszczyc (2018) compare voluntary-pay auctions - in which the auctioneer asks all bidders to pay their own highest bid - and winner-pay auctions both in charity and non-charity settings. They find the voluntary-pay auction format raises relatively higher revenue than the winner-pay format for charity auctions.

Researchers even have done comparisons within a single auction format, such as, the comparison between different number of prizes in a single auction format (Faravelli and Stanca, 2012). Faravelli and Stanca (2012) compare single versus multiple-prize in all-pay auction format. They find for a given total prize sum, a single large prize generates higher contributions to the public good than three smaller prizes. Another example is the comparison within all-pay auction format. Orzen (2008) and Goeree et al. (2005) both find the last price all-pay auction format, in which the bidder who bid highest wins and all bidders pay the lowest bid, does better than first price all-pay auction.

There has been research on the motivations of bidders in charity auctions as well. One reason why bidders in charity auctions bid more is that bidders derive utility from charity donations (Elfenbein and McManus, 2010; Halfpenny, 1999; Haruvy and Leszcayc, 2009; Leszczyc and Rothkopf, 2010). Leszczyc and Rothkopf (2010) show that the ending price in charity auctions are significantly higher than the ending price in non-charity auctions because the charitable motives of bidders. Haruvy and Leszcayc (2009) find warm glow and pure altruism two motives in their field experiment involving simultaneous pairs of auctions that are identical in all respects but percentage of the proceeds donated to charity. The above mentioned utility may be from concerns about the total money that can be collected by the charity. This type of motive is always referred to as pure altruism (Elfenbein and McManus, 2010; Halfpenny, 1999; Haruvy and Leszcayc, 2009; Leszczyc and Rothkopf, 2010). The utility also can be derived from the act of giving, which is the impure altruism motive (Andreoni, 1989; Elfenbein and McManus, 2010; Haruvy and Leszcayc, 2009; Leszczyc and Rothkopf, 2010). Bidders derive utility from self-image and other sources as well (Ariely et al., 2009).

Unlike the past studies in this topic, we focus on testing the existence of some specific motives and the strength of them in charity auctions. We also reveal the distribution of public goods valuations by comparing the valuations bidders placed on identical products auctioned simultaneously in charity and non-charity auctions on eBay.

2.2 Structural analysis of auction data

Direct Methods Structural analysis of auction data begins with Paarsch (1992). Paarsch (1992) uses generalized method of moments (GMM) estimation to estimate two simple parametric models with one within a common value setting and the other within a private value setting. Then he applies these framework to data from tree planting contract auctions to decide between the two competing paradigms. After him, there were a few researchers using this direct method, such as Donald and Paarsch (1992), Laffont et al. (1995), and Laffont et al. (1993). For all these studies, because of the complexity associated with the computation of the Bayesian Nash equilibrium strategy, they mainly hinge upon specific parametric specifications of the bidders' private values distribution.

Indirect Methods Since Guerre et al. (2000a) provide a natural structural way to reveal the distribution of valuations in a nonparametric framework using the bid - value relationship in first-price auctions within the independent private value (IPV) paradigm, numerous studies have been done on nonparametric structural estimation of auctions in IPV paradigm, such as Hickman and Hubbard (2015), Lee et al. (2004), Henderson et al. (2012), and Athey and Haile (2002). Majority of these research on nonparametric structural estimation for auctions widely use rule-of-thumb to select the bandwidth in the auction literature, which attempts to provide an optimal bandwidth for the bid density, except Henderson et al. (2012). Henderson et al. (2012) provide a framework for automated selection of the bandwidth for the bid - value relationship in first-price auctions.

3 Theory

3.1 Second-price auction without charity

We consider an economy with n symmetric risk-neutral bidders. Bidder i's private valuation on auction item v_i^p is private information, which is drawn independently from an identical distribution (i.i.d) with $F(v^p)$ as the cumulative distribution function with support $[\underline{v^p}, \bar{v^p}]$ and $f(v^p)$ as the probability density function. The distribution of private values ($F(v^p)$) is known to all the bidders, which makes the bidders symmetric. The bidder who bids the highest wins the auction and pays the second highest bid. Let x be the highest of the other (n-1) bidders, then the distribution of x is $F(x)^{n-1}$. An individual bidder i who assumes everyone else is bidding according to the increasing bidding strategy $b(v^p)$ is assumed to use his private value information to choose a bid b_i to maximize his expected payoff. If $b_i > b(x)$, bidder i wins the auction and pay $b(x)$ with payoff $v_i^p - b(x)$; if $b_i < b(x)$, bidder i losses the auction and pays zero with zero-payoff. In summary, the payoff function in a second-price auction without charity is as follows,

$$p_i = \begin{cases} v_i^p - b(x), & if\ i\ wins \\ 0 & otherwise \end{cases}$$

We assume the equilibrium bid function, b = b(v), is continuous, increasing, and differentiable, which proves the existence of inverse $\phi(b)$. Then bidder i's expected payoff from bidding b_i is

$$\pi_i(b_i; v_i^p) = \int_{\underline{v^p}}^{\phi(b_i)} [v_i^p - b(x)] dF(x)^{n-1} \tag{1}$$

In equation (1), $v_i - b(x)$ is bidder i's payoff if he wins, $\int_{\underline{v}}^{\phi(b_i)} dF(x)^{n-1}$ is the probability bidder i will win the auction if he bids b_i. With some algebra, equation (1) can be rewritten as

$$\pi_i(b_i; v_i^p) = \int_{\underline{v^p}}^{\phi(b_i)} [v_i^p - b(x)] f(x)^{n-1} dx \tag{2}$$

From the above payoff function, an individual bidder's utility comes from the private consumption of the auction item when he wins, otherwise his utility is zero. According to Vickrey (1961), an individual bidder bids his true value in a second-price auction without charity. Let v_i^p denote bidder i's valuation of the auction item. Then bidder i's bidding function is:

$$v_i^p = b_i \tag{3}$$

Let $G(b)$ and $g(b)$ be the cumulative distribution and density functions of the bids, respectively. The estimation of $G(b)$ and $g(b)$ hinge critically upon the information we have. Consequently, two scenarios with different information on bids are considered in the next two subsections.

3.1.1 If we know all the bids

If we know all the bids, the distribution and density of private values will be reaped with the following kernel estimation. I first estimate the $G(b)$ and $g(b)$ nonparametrically using the below two formulas.

$$\hat{G(b)} = \frac{1}{h}\sum_{i=1}^{N}\sum_{t=1}^{T_i} K(\frac{b - b_{it}}{h}) \tag{4}$$

$$\hat{g(b)} = \frac{1}{h}\sum_{i=1}^{N}\sum_{t=1}^{T_i} K(\frac{b - b_{it}}{h}) \tag{5}$$

where $K(\cdot)$ is a kernel function required to be non-negative, bounded, and symmetric density function with compact support. h is the bandwidth required to converge to zero as T goes to infinity. We use Gaussian kernel and bandwidth that chosen by the adaptive rule-of-thumb in this paper. Using $v_i^p = b_i$, we then determine $F(v^p)$ and $f(v^p)$, which correspond to $G(b)$ and $g(b)$ respectively.

3.1.2 If we only know the winning bids

The distribution of bids is necessary to explore the distribution of private values according to the equation (3). To figure out the distribution of bids, we first estimate the distribution and density of winning bids, $\hat{G_w}$ and $\hat{g_w}$ using equations (4) and (5). Having the $\hat{G_w}$ and $\hat{g_w}$ in place, we compute the corresponding distribution and density of bids using the $(n-1)$ order statistic because in a second-price auction, the payment is the second-highest bid.[6] Last, we determine the $F(v)$ and $f(v)$, which are $G(b)$ and $g(b)$ respectively since $v_i^p = b_i$.

In conclusion, for either case, it is feasible to generate the distribution of bids,$\hat{G(\cdot)}$, and corresponding density,$\hat{g(\cdot)}$, by following the estimation methods outlined above. As a result, it becomes feasible to deduce bidders' private valuations through the equilibrium mapping between valuations and bids. This method allows us to estimate the distribution of private values in second-price auctions that do not involve charity.

3.2 Second-price auction with charity

In charity auctions, previous studies demonstrate that bidders are often willing to pay more for items linked to charity. This heightened willingness arises from the fact that winning such items contributes to charitable donations. Additionally, a distinctive characteristic of charity auctions is the generation of extra utility for every bidder. Remarkably, this added benefit is independent of whether the bidder wins or loses the auction. Specifically, if they win the auctions, they give their own money to charities and derive utility from increasing the money collected by charities. While if they lose, they still gain utility from the money collected by charities from other participants in these auctions (Tan, 2020). Therefore, a bidder in a charity auction may receive both valuation of the auction item itself and the public valuation from the money that goes to a charity. We denote the valuation bidder i gets from the auction item as v_i^p and the valuation from charity/public good value as v_i^c. We

[6] The distribution of all the bids, $\hat{G}$, is calculated from $\hat{G_w} = n * \hat{G_w}^{n-1} - (n-1) * \hat{G_w}^{n}$; The density of all the bids,$\hat{g}$, is calculated from $\hat{g_w} = n * (n-1) * \hat{G}^{n-2} * [1-\hat{G}] * \hat{g}$.

define this joint value of auction item and charity as full private value in charity auctions.

Consequently, depending on how bidders treat the money collected by charities from themselves and from others, the motivation of higher-bid in charity auctions can be decomposed to the result of their concern of the total amount of money collected by a charity (pure altruism), or their enjoyment of the act of giving itself (warm-glow), or both. Meanwhile, the estimation of the distribution of joint private value in charity auctions essentially hinges upon the identification of motivations of giving. On the one hand, if bidders place higher bids in charity auctions out of altruism, they obtain additional value beyond the auction item from charity, regardless of whether they win the auction or not. The intuition behind this is that such bidders offer higher bids in a charity auction compared to a non-charity auction because they derive utility from the charity, irrespective of the source of the donation. In other words, these bidders regard donations to a charity made by themselves and others as equally beneficial. Consequently, the challenge for bidder i's is to choose his bid, b_i to maximize his expected payoff, assuming that all other participants in the auction use the same bidding strategy.

On the other hand, certain bidders distinguish between donations made by themselves and those made by others. Specifically, they place a higher value on their own act of giving, suggesting a unique intrinsic satisfaction derived from personal donation. This difference in valuation, known as the 'warm glow' effect, manifests more strongly in their own charitable actions. Given that these two motivations result in different full private values for bidders in charity auctions, it becomes necessary to estimate altruistic parameters. This estimation process is discussed in the following subsection.

Charity Auction Preliminaries We consider charity auctions that sell identical auction items as in the auctions without charity in Section 3.1. In each of these charity auctions, the seller possesses this item to sell at a second-price auction and donates a percentage (θ) of the transaction price to an individual charity organization. Even when considering an identical auction item, we cannot expect all bidders to place the same bids as in non-charity auctions. This is because charitable motivations play a significant role in influencing a bidder's behavior in charity auctions. Bidders with no charitable motivation will bid exactly the same as in non-charity auctions. However, if bidders have positive charitable motivations, their bids in charity auctions are expected to surpass those in non-charity auctions. This is because, apart from the utility derived from the auction items, they also derive additional benefits from the act of charity, regardless of whether they win the auctions or not. We assume that there are no revenue-enhancing strategies, such as minimum bids and entry fees. There are $n \geq 2$ risk-neutral bidders whose private valuations of the auctioned item, v^p draw independently from the distribution F with support $[\underline{v^p}, \bar{v^p}]$ as in the non-charity auctions. Bidder i's private value on charity v_i^c is private information, which is drawn independently from an identical distribution with $H(v^c)$ as the cumulative distribution with support $[\underline{v^c}, \bar{v^c}]$ and $h(v^c)$ as the probability density function. Bidders' valuations for both the auctioned item and the linked charity are considered private information, but the distributions F and H are common knowledge. Let F and H be differentiable on the interior of their supports with positive density f and h.

Following Engers and McManus (2007), we specify that the return to bidder i from the host charity collecting one dollar from i is θ, while the return to i from another bidder's dollar being transferred to the charity is λ. To allow for a warm glow from charitable giving, we specify that $0 \leq \lambda \leq \theta < 1$. The θ is restricted to be less than one because values of $\theta \geq 1$ render the idea of a charity auction moot since i wishes to transfer all of her money to the charity. If $\theta = \lambda$, the return from participating in the auction is purely altruistic since bidder i's benefit from increasing the charity's output is independent of the source of the money. When $\theta > \lambda$, there is an additional, warm-glow to bidder i when it is his own money that goes to the charity.

Bidding in a Charity Auction In this subsection, we will derive the equilibrium bid function for the charity auction following closely from Engers and McManus (2007). In a second-price charity auction, bidder i's payoff depends on her true value on both the auction item and associated charity, her bid and the highest and the second-highest bids submitted by the other participants in the auction. Suppose x is a random variable that may represent the highest or second-highest of the other $n-1$ participants' valuations. So, if x is the highest, the distribution function is $F(x)^{(n-1)}$, and if x is the second-highest, the distribution function is $\{(n-1)F(x)^{(n-2)} - (n-2)F(x)^{(n-1)}\}$. All bidders use the bid function B to map their valuations for the auctioned item into bids. We assume this bid function is continuous, increasing, and differentiable, then the inverse $\phi(b)$ exists. There are three possibilities for an individual bidder i in a second-price charity auction. First, bidder i bids the highest, so she wins the auction with paying the highest bid submitted by the other $(n-1)$ participants. In return for her payment, she receives the auctioned item and a benefit of θ for each one dollar of her payment that goes to charity (i.e. δ times her payment) , where δ is the percent of selling price that will go to charity. Second, bidder i submits a second-highest bid with the probability $(n-1)F(\phi(b_i))^{(n-2)}[1-F(\phi(b_i))]$. Then she losses the auction, but she still benefits from it because her bid decided the price of the auction item. And the amount of benefit is $\delta\lambda B(b_i)$. Third, if i bids lower than the first and second-highest bids of the other $(n-1)$ bidders, she losses the auction and benefits from the winner's payment at $\delta\lambda B(x)$. Combining these three cases, we can write bidder i's expected return from auction as if she bids $b_i = B(v_i)$:

$$\begin{aligned}\pi(b_i \mid v_i^p, n) = \int_{\underline{v}}^{\phi(b_i)} [v - B(x) + \delta\theta B(x)] dF(x)^{n-1} \\ +\delta\lambda(n-1)F(\phi(b_i))^{(n-2)}[1-F(\phi(b_i))]B(b_i) \\ +\delta\lambda \int_{\phi(b_i)}^{\bar{v}} B(x)(n-1)(n-2)F(x)^{n-3}[1-F(x)]dF(x)\end{aligned} \tag{6}$$

Applying the first-order condition for an incentive-compatible selection of b_i and some algebra,

$$\frac{d\pi(b_i \mid v, n)}{db_i} \mid_{b_i = v_i^p} = 0$$

we obtain our bid function for a second-price charity auction::

$$B(v_i^p) = (\frac{1}{1 - \delta(\theta - \lambda)})\{v_i^p + \int_{v_i^p}^{\bar{v}} (\frac{1 - F(x)}{1 - F(v_i)})^{\beta} dx\} \tag{7}$$

where β denotes $\frac{1-\delta(\theta-\lambda)}{\delta\lambda}$. From this bid function, we can see it is independent of the number of bidders,n. Also, when θ and λ are zero, B(v)=v, which is truthful bidding in the classic textbook second-price auction with non-charitable bidders.

Identification In this subsection, we are going to show how to identify both the distribution of valuation, $F(v)$, and altruistic preferences, θ and λ. After some algebra,[7] equation (7) can be rewritten as

$$v_i^p = b_i(1 - \delta(\theta - \lambda)) - \frac{1 - F(v_i^p)}{(\beta + 1) f(v_i^p)} \tag{8}$$

Let G(b) and g(b) denote the cumulative distribution and density function of the bids, respectively. Since $G(b) = F(\phi(b))$ and $g(b) = f(\phi(b))\phi\prime(b)$,[8] the equation (8) can be rewritten as

$$v_i^p = b_i(1 - \delta(\theta - \lambda)) - \frac{1 - G(b_i)}{(\beta + 1) g(b_i)} \tag{9}$$

In equation (9), v_i^p at the left-hand side, θ and λ at the right-hand side are the unknowns.

To back out the v_i^p in equation (9), estimations of $g(\hat{b})$, $G(\hat{b})$, θ and λ are necessary. We use the same way in the second-price auction without charity to generate $g(\hat{b})$ and $G(\hat{b})$. To identify θ and λ, we follow Fioretti (2020) relying on the variation in the percentages, δ, donated across charity auctions. The basic intuition behind this method is that the right-

[7]More detail can be found in Appendix B.

[8]Since $G(b) = Pr(b(v) < b) \equiv Pr(v < \phi(b)) = F(\phi(b))$.

hand side of equation (9) is strictly increasing and differentiable in b_i, implying that no two different combinations of θ and λ will yield the same vector of pseudo-private valuations given δ. Furthermore, apart from the percentage of proceeds donated to charity (i.e., charity auctions donate a non-zero specific percentage while non-charity auctions donate nothing), charity and non-charity auctions are essentially identical. Coupled with the assumption that in both charity and non-charity auctions, bidders' private valuations for auctioned items are independently and identically distributed (i.i.d), Theorem 1 from Guerre et al. (2000b) suggests that if a unique distribution of private valuations for the auction items exists, it should hold true across both types of auctions. Therefore, if there are two sets of charity auctions with different donation percentages, δ_A and δ_B, otherwise they are identical, their distributions of private valuations should be the same under the true altruism preference combination $\{\theta, \lambda\}$. That is, $F(\cdot|\theta, \lambda, \delta_A) = F(\cdot|\theta, \lambda, \delta_B)$, which we use to identify $\{\theta, \lambda\}$.[9] Once we got $\{\hat{\theta}, \hat{\lambda}\}$, we plug them back to equation (9), we can get the corresponding pseudo-private valuations on the auction items. Adding the pseudo-private valuations on auction items up to the valuations on the charities, θ, we obtain the joint private valuations in charity auctions and thus the distribution of the joint valuation $H(V^j)$.

Specifically, there are six steps to identify $H(V^j)$. First, we control the heterogeneity across charity auctions to get homogeneous charity bids. Second, generate $\hat{G(b)}$ and $\hat{g(b)}$ by the same methods in non-charity auctions; Specifically, we nonparametrically estimate $\hat{G(b_w)}$ and $\hat{g(b_w)}$ using Gaussian kernel and adaptive rule-of-thumb for bandwidth selection and then use the $(n-1)$ order statistic to obtain $\hat{G(b)}$ and $\hat{g(b)}$. Third, following Fioretti (2020), we build the FOCs as in equation (9) for both sets of charity auctions. Then we match each FOC in charity auction with δ_A to a FOC of the charity auction with δ_B along the quantiles of the distributions of bids, i.e. FOCs with $\delta_A(b^{\delta_A})$ and $\delta_B(b^{\delta_B})$ are matched if $\hat{G}(b^{\delta_A}; \delta_A) = \hat{G}(b^{\delta_B}; \delta_B)$. For the matched FOCs, their corresponding pseudo-private valuations in the left-hand side of FOCs will be equal under the true value of altruism preference combination

[9] The same method can also be found in (Lu and Perrigne, 2008).

$\{\theta_0, \lambda_0\}$. That is, $v_\tau^{\delta_A}(\theta_0, \lambda_0) = v_\tau^{\delta_B}(\theta_0, \lambda_0)$. Thus, the criterion function is

$$\min_{\theta,\lambda} \quad \tfrac{1}{T}\textstyle\sum_{\tau}^{T}(v_\tau^{\delta_A}(\theta_0, \lambda_0) - v_\tau^{\delta_B}(\theta_0, \lambda_0))^2 \tag{10}$$

where T is the number of quantiles we chose for both sets of charity auctions with δ_A and δ_B. Fourth, reveal $\hat{v^p}$ at the left-hand side of equation (9) for both sets of charity auctions by plugging $\{\hat{\theta}, \hat{\lambda}\}$ got in the third step. Fifthly, back out the distribution of private values $F(\cdot)$ using $\hat{v^p}$ obtained in the fourth step. Lastly, compute the total private value in each charity auction using $v_i^p + \delta\theta B(x)$ and the distribution of total value $H(\cdot)$, where $B(x)$ is the winning bid or the highest bid by the $(n-1)$ bidders.

3.3 Back out the valuation of charity from the difference between joint valuation and private valuation

In this section, we back out the valuation of charity from the difference between the joint valuations in charity auctions and the valuations of the auction items own in non-charity auctions. By comparing the estimated distribution of private valuations in charity auctions with the the estimated distribution of private valuations in non-charity auctions, we expect the difference is the distribution of valuations of charity given the bidders in both charity and non-charity auctions have the same underlying distributions of the auction item own valuations. The behind intuition is that an individual bidder has the same underlying private value on the auction item. Thus, we can treat the difference between his bids in a matched pair charity and non-charity auctions as his private value on the charity linked to the charity auction.

We follow the notation and terms in Poe et al. (2005) to estimate the difference of these two empirical estimated distribution we obtained in previous sections, $f(\hat{v^p})$ and $h(\hat{v^J})$. Let $\hat{V^p}$ and $\hat{V^J}$ are the two estimated parameters and be denoted by X and Y and be depicted by the vectors $\hat{\mathbf{x}} = (x_1, x_2, \ldots, x_m)$ and $\hat{\mathbf{y}} = (y_1, y_2, \ldots, y_n)$ with corresponding probability distribution functions $f_{v^p}\hat{(}v^p)$ and $h_{v^J}\hat{(}v^J)$. In our case, the two empirical distribution are estimated independently and discrete. So, the probability of x_i in $\hat{\mathbf{x}}$ is $1/m$, similarly, the probability of y_j in $\hat{\mathbf{y}}$ is $1/n$.

We use the complete combinatorial method suggested in Poe et al. (2005) as the one which provides an exact measure of the difference of two distributions for independent samples. In this approach, we calculate every possible difference between these two distributions. Using the previous notation, then the empirical distribution of the difference $\hat{Z} = \hat{X} - \hat{Y}$ is given by:

$$\hat{X}_i - \hat{Y}_j = \hat{X}_i + (-\hat{Y}_j) \tag{11}$$

$\forall i = 1, 2, \ldots, m; j = 1, 2, \ldots, n.$

where each difference is placed on equal weight $1/(m * n)$.

A potential threat to the above identification would be if the underlying distribution of the private values on the auction item from charity auction set differed from it in the non-charity auction set given not everyone who participated in auctions in charity auction set shows up in the non-charity auction set as well. While having the assumption 1 and the proof that our data satisfies this assumption (in Section 7.1.3) in place, it is safe to employ this method at here.

3.4 The underlying assumption of this method

The above method only works when bidders' private values on the auction item, v are independent on donation percentage, δ. That is, there is no phenomenon that high valued bidders systematically join auctions with higher percentage of price donated, or it is the other way around. So the following assumption is necessary in this estimation method:

Assumption 1: $F(\cdot)$ does not depend on donation percentage, δ.

Theoretically, this assumption is satisfied in our model because in our model preliminaries in

Section 3.2, we assume this model is under the independent private value framework. Practically, we prove that this assumption holds in our data by checking whether the distribution of bidders' private values on the auction items in charity auction set is the same as it in the noncharity auction set.

4 Auction Data

The data set is publicly available and is from Elfenbein and McManus (2010). The original matched charity and non-charity data is collected from eBay auctions that closed between March and December 2006. The auction data set includes product categories that are most likely to have matched simultaneous charity and non-charity auctions on eBay. Matched charity and non-charity auctions refer to those auctions where the items up for bid are identical in their physical characteristics. Specifically, it includes consumer electronics, cameras and photography equipment, DVDs, computer equipment, and gift certificates.

The matching procedure between charity and non-charity auctions is as follows. Elfenbein and McManus (2010) began with a charity auction on Giving Works concluded with a sale, and then searched up to five non-charity auctions ended in a sale within five days of the Giving Works auction item. This procedure was specifically designed to ensure that pairs of matched charity and non-charity auctions were concurrently active, given that the majority of eBay auctions typically span a seven-day period. If more than five matched non-charity auctions are found, they choose the five ones that are closest to the charity auction's ending time. For more details on the matching process, please see Elfenbein and McManus (2010).

In total, Elfenbein and McManus (2010) collected 2433 auctions with 723 charity auctions, resulting in an average around 2.4 non-charity auctions matched to per charity auction. For each auction, the data includes the winning bid (i.e. price), the shipping fee, the date and the time of the bid, winning bid's associated bidder's user name and feedback score, and the number of bids. In each charity auction, data also includes donation percentage and the name of the charity. The detail of the summary statistics for the data can be found in Table 1 in Elfenbein and McManus (2010). In this paper, the variable of most interest is the sale price including shipping, which is the summation of closing price and the shipping fee charged by the seller in each auction.

Table 1 shows the summary statistics of all the variables we used in this paper. As Table 1 shows, the average prices (including shipping fee) in non-charity and charity auctions are \$95.78 and \$102.38 respectively. Charity auctions run longer than non-charity ones on average. The donation percentages in charity auctions are most often 100 and 10. Over 59% (i.e., 429/723) auctions donate 100 percent to charities. Over 29% (i.e., 211/723) auctions donate 10 percent to charities. Figure 1 shows the histogram of donation percentage.

5 Monte Carlo Simulation

In this section, we run a Monte Carlo simulation with a twofold goal. The first one is to verify whether we can obtain the consistent estimate of altruistic parameters in the charity auction set using the variation in the donation percentages. The other one is to investigate whether we can deduce the consistent underlying distribution of private values solely from the observed winning bids in both the charity and non-charity auction sets.

Setting up There are charity and non-charity auctions. They are fundamentally identical, with the exception of their charitable contributions. In charity auctions, all or part of the transaction prices are donated to charitable causes, while non-charity auctions do not involve such donations. In the charity auctions, we assume there are two sets of auctions, which we refer to as set A and B respectively. There are 1000 auctions in each of these two auction sets, which defines the sample size for both auction sets. The two charity auction sets are identical with the only difference being that they donate different percentages of their winning bids (henceforth, we refer it as price) to charities. Specifically, in set A, they will donate 10% of their prices to charities, while 85% of their prices will be given to the charities in set B. In both auction sets, there are 2 bidders in each auction and the private value for the auction item for all bidders is drawn from a normal distribution on (30,3).[10] All the bidders in each auction set bid according to equation (7). We define the true altruistic parameters as 0.75 and 0.25 respectively. The Monte Carlo simulation is carried out according to the following steps.

Procure to estimate altruistic parameters 1. In auction set A, draw private values from F for all bidders in each auction. In total 1000*2 values have been drawn because there are 2 bidders in each auction.

2. In auction set A, for each private value obtained in step 1, calculate all the corresponding bids according to equation (7). In total there are 1000*2 bids. Find out the second highest bid in each auction, which is the winning bid for each auction.

3. Repeat step 1 and 2 for auction set B.

4. Nonparametrically estimate the distribution and density of winning bids in both auction sets A and B. We use Gaussian kernel and adaptive rule-of-thumb bandwidth proposed by Silverman (2018)(Equation(3.29)).[11] I trim the data following Guerre et al. (2000b) who suggested trimming observations close to 0.5 $\times$ bandwidth to the boundary. This is also used in Fioretti (2020).

5. Given the number of bidders in each auction, we can use the relationship between the distribution and density of winning bids,G_w, g_w and the distribution and density of all bids, G, g to find out the distribution of all bids using the distribution of winning bids that we obtain in the step 4 in both auction sets A and B.[12]

6. Match the quantiles of auction sets A and B according equation (9). Specifically, if $G^A(b^A) = G^B(b^B)$, we say they are matched.

[10] We also use an uniform distribution on [0,1]. The simulation result is reported in Appendix C.

[11] The specific formula for the bandwidth is $1.06 * \delta * n^{-1/5}$ for density estimate and $1.06 * \delta * n^{-1/4}$ for distribution estimate. Where, δ is defined as min(standard deviation, interquartile range/1.349).

[12] The relationship between distribution and density of winning bids and the distribution and density of all bids are $\hat{G_w} = n * \hat{G_w}^{n-1} - (n-1) * \hat{G_w}^{n}$ and $\hat{g_w} = n * (n-1) * \hat{G}^{n-2} * [1-\hat{G}] * \hat{g}$ according the order statistic.

7. Find and save the parameters $(\hat{\theta}, \hat{\lambda})$ that minimize the criterion function (10) in section 3.2.

8. Repeat step 1 to 7 a large number of times.

Estimated Result We repeat this procure 400 times and the estimated result is {0.735,0.239}. The differences between the initial true and the estimated parameters are less than 2%. We also tried different initial pairs and the estimates are reported in Table 2. In Table 2, we find that the difference between the true and estimated altruistic parameters in all pairs are within 4% implying that our approach reaps consistent estimates of altruistic parameters.

Comparison between the true and the estimated distributions of bids in charity and non-charity auctions In both charity and non-charity auction sets, see Figure 2 and Figure 3, the true and estimated distributions of private values are almost overlapped. We also run a Kolmogorov–Smirnov (KS) test ($p = 0.00$) implying the estimated distributions of private values are statistically the same as the true underlying distribution of private values in both charity and non-charity auction sets. This result of equality-test combined with the comparison between estimated and true private value distributions supports that this approach can achieve consistent estimate of the underlying private value distribution using the winning bids only.

6 Concerns in the above identification method

Before we apply the aforementioned identification method to our field data, we address some potential concerns related to this method. Specifically, in this section, we replicate some of the main results from Elfenbein and McManus (2010). We lay the groundwork for the structural identification in subsequent sections by illustrating how the donation percentage impacts charity premium and the number of bidders in charity auctions.

We first replicate the impact of donation on the price by running the following product-specific fixed effects model:[13]

$$log(\mathbf{Price_{im}}) = \alpha_m + \mathbf{Donation_i}\beta_1 + \mathbf{Auction_i}\beta_2 + \mathbf{Seller_i}\beta_3 + \epsilon_{im} \qquad (12)$$

where $\mathbf{Price}_{im}$ is the sum of price and the shipping fee. Auction length and the buy-it-now dummy variables are in the **Auction** vector. Either donation dummy or different donation percentage dummies are included in the vector of **Donation** across specifications.[14] More specifically, we utilize the donation dummy when replicating the overall charity premium, whereas we employ the different donation percentage dummies when replicating the premiums across various donation percentages. In the vector **Seller**, we include seller's rating and dummy variable for individual seller's power seller's status.[15] The heterogeneity across auction items, it is captured by the product-specific fixed effects, α_m. The results are reported in Table 1 in Appendix D. We successfully replicated the original finding that bidders pay, on average 6% higher in charity auctions than the matched noncharity auctions.

Second, we replicate the heterogeneity in charity premiums across donation percentages. We employ the same product-specific fixed effects model in equation (12) and the same variables in vectors **Auction** and **Seller**. While in the vector **Donation**, we use donation percentage dummies here. The results are reported in the column 1 in Table 1 in Appendix D. According to the table, in 10% and 100% auction sets, bidders bid 5.4% and 7.3% higher,

[13]In the spirit of Elfenbein and McManus (2010), we treat the auction item in each matched pair auctions as a different product. In total, there are 723 different products in our data set. The evidences that each match is a different product from the original paper are as follows. First, when they report the results from the fixed-effect model (equation (1)) in Table 3, they mentioned that the models contain a fixed effect for each product (p. 44). Then, when they use this model again for results in Table 6 in page 48, they use the language "We continue to employ match-specific fixed effects.". These two points imply that product and match are two exchangeable notations in the original paper.

[14]The donation dummy is 1 if the donation percentage is not zero, otherwise is 0. The donation percentage dummies include three dummies: 10%- SHARE, 100%-SHARE, and MID-SHARE.

[15](Elfenbein and McManus, 2010, pp.40) define power sellers as sellers who complete more than $1,000 per month in eBay transactions, maintain 98 percent positive feedback, and have an overall rating that exceeds 100.

respectively, than in the matched non-charity auctions. These results align closely with the findings presented in the original paper. In the origin paper, they find that the estimated premium are 5.1% and 7.2% in the 10% and 100% auction sets when shipping fee is included. Based on our replicated results, we get the same conclusion as the original paper that bidders bid significantly higher in both 10% and 100% auction sets than the corresponding matched non-charity auction sets. Additionally, we ascertain that there's no statistical difference between these two premiums, as per the results of a KS test ($p = 1$). This finding suggests that the selection of bidders into auctions isn't determined by the size of the premium. In other words, we find no evidence to suggest that bidders with higher (or lower) premiums systematically gravitate towards charity auctions with correspondingly higher (or lower) donation percentages. When the donation percentages are between 10 and 100, bidders pay about 3% higher in charity auctions than the matched noncharity auctions, which is consistent with the original finding (i.s. 2.8%).

Last, to replicate the result that there is no bidder selection across charity and noncharity auctions, we first replicate it involving a similar specification in (12). But this time, we have the number of bidders in **Auction** as a control. The result, which is reported in the column 2 in the Table 1 in Appendix D, shows that the coefficients on 10%-SHARE and 100%-SHARE are nearly unchanged relative to column 1 leading to the original finding that the number of bidders has no significant impact on the charity premium. We then use a subsample of the bids placed by bidders who were active both in the charity and noncharity auctions within a matched set of sales. Due to data limitation, we use ordinary least squares (OLS) for the below specification:

$$log(\mathbf{Bid_{im}}) = \mathbf{Donation_i}\beta_\mathbf{1} + \mathbf{Bidder_i}\beta_\mathbf{2} + \epsilon_{im} \tag{13}$$

where $\mathbf{Bid_{im}}$ is the bidder i's bid amount in auction m. In vector **Donation**, we include the donation dummy. In vector **Bidder**, we include the log of bidders' feedback. The result, which is in the column 3 in Table 1 in Appendix D, shows that an individual bidder bids around 7% higher for the same auction object in a charity auction than in the matched noncharity auction(s). This result combined with the impact of the number of bidders on the charity premium provides some evidence that the charity premium is derived by an increase in bidders' willingness to pay for the charity-linked item rather than bidders selection across charity and noncharity auctions. Having addressed the aforementioned concerns with our data, we apply the above identification to our eBay auction data in the next section.

7 Empirical Analysis

7.1 Estimate of Altruistic Parameters

7.1.1 Estimation Procure

We estimate the altruistic parameters $\{\theta, \lambda\}$ by using the variation in donation percentage, δ, across charity auctions. To ensure accurate estimation, we select two distinct sets of donation percentages. It has been demonstrated by Fioretti (2020) that when the two sets of donation percentages are too close to each other, the estimation of the altruistic parameters may fail in a Monte Carlo simulation. In our dataset, the most frequently observed donation percentages are {10%, 100%}, as shown in Figure 1. Hence, we choose these two sets of donation percentages for estimating $\{\theta, \lambda\}$. For simplicity, we refer the two groups of charity auctions as "Charity Set A" and "Charity Set B," representing the "10% auctions" and "100% auctions," respectively. The estimation process consists of four steps, which are outlined as follows.

Step 1 In matched pairs of charity and non-charity auctions, the auctions are identical in every aspect except for the charity element. In the charity auction, a portion of the price goes to a charitable cause, while its paired non-charity counterpart doesn't make any charity contributions. It's important to note, however, that differences may exist between different matched pairs. The difference includes different auction items, different auctioneers' reputations, different active bidders, different number of bids, different auction lengths, different time and date of auctions, different shipping charges, and different charities. To get homogeneous auction pairs, we run the product-specific fixed effects model as in equation (12) with donation percentage dummies in vector **Donation**. We take the $exp^{\epsilon_{im}}$ plus the coefficient of 10% as the homogeneous winning bids from the charity set A, and the $exp^{\epsilon_{im}}$ plus the coefficient of 100% as the homogeneous winning bids from the charity set B.

Step 2 We nonparametrically estimate the distribution and density of wining bids for both sets A and B, represented as $\hat{G_w}$ and $\hat{g_w}$ respectively. Here, we use Gaussian kernel and a bandwidth chosen according to the adaptive rule-of-thumb. Specifically, we use $1.06*\Delta*n^{-1/5}$ for density estimate, and $1.06 * \Delta * n^{-1/4}$ for distribution estimate, where, Δ is defined as min(standard deviation, interquartile range/1.349).

Step 3 For each set, we compute the distribution and density of bids based on the distribution and density of winning bids derived from Step 2 and the order statistic. Since the fact that the winning bid in a second-price auction is the second highest bid, we use the $(n-1)$ order statistic, where n is the number of bidders. Specifically, the distribution of bids, $\hat{G}$, is calculated from $\hat{G_w} = n * \hat{G_w}^{n-1} - (n-1) * \hat{G_w}^{n}$. Similarly, the density of all the bids, $\hat{g}$, is calculated from $\hat{g_w} = n * (n-1) * \hat{G}^{n-2} * (1-\hat{G}) * \hat{g}$.

Step 4 We match the equation(9) in set A to it in set B according to the quantiles of the distributions of bids. That is, the equation(9) in set A is matched to it in set B if $\hat{G}^A(b^{10\%}, \delta = 10\%) = \hat{G}^B(b^{100\%}, \delta = 100\%)$. By this way, the left hand side of matched equation(9) would be equal because set A and B have the same distribution of private values based on the Assumption 1. The same underlying private value distribution implies that at each quantile, $\tau \in [0,1]$, $v_\tau^{10\%} = v_\tau^{100\%}$. Here, to avoid the boundary issue in nonparametrical estimation, we use $\tau \in [0.05, 0.95]$ and as a return we obtain 164 different numbers of τ.[16] Then, we plug these 164 matched pairs of equation(9) into the critical equation (10) to estimate $\{\theta, \lambda\}$.

7.1.2 Estimation Results

Applying the above approach, we find that the estimated value for θ is 0.09, while the estimated value for λ is close to zero. That is, for an individual bidder i, the return of each-dollar donation from her is 9 cents, while the return to her from an one-dollar giving from other givers is close to zero. Since an individual giver i thinks the giving from herself brings her more happiness, warm glow results. The magnitude of warm glow is the difference in the return of an one-dollar donation from giver i and from other givers. Consequentially, in our data, the warm glow is 0.09 (i.e.$\theta - \lambda = 0.09 - 0.00$).

Using the magnitude of warm glow, we can explore the total amount of return from giving in our data. For example, in auctions with 100% donation, the average price is $67.72, which is the auction winner's average payment. As a return for her payments, the winner gets about $6.09 (i.e.$67.72*0.09) back in total. However, for bidders who lose in these auctions, they almost get nothing from winners' payments because their valuation of others' payments is low (λ is pretty close to zero) in these charity auctions. In each of these 100% donation charity auctions, the total amount of donation is $67.72 regardless who pay for it. However, from the same amount of giving, auction winners and losing bidders derive different returns from the charity. Auction winners receive more returns from the charity compared to losing bidders. This implies that bidders evaluate their own giving to charities more than the giving from others. In other words, bidders indeed derive a valuation from the act of giving itself, a phenomenon referred to as the warm glow (Andreoni, 1989). In our example, the total return from the warm glow, on average, is $6.09 (i.e.$6.09 - $0.00).

[16] 164 is the number of charity auctions donating 10% to charities. The number of charity auction donation 100% is 375.

7.1.3 Robustness Tests

As we mentioned in Section 3.3, the above estimation method relies on Assumption 1. To investigate whether our data set satisfies this assumption and to assess the robustness of our results, we prove that Assumption 1 holds in our data in this subsection.

Proof.

Step 1 We obtain the homogeneous winning bids in both charity and non-charity auction sets using the specification in 12 with a donation dummy in the vector **Donation**.

Step 2 We nonparametrically estimate the distribution and density of winning bids with Gaussian kernels and bandwidths chosen according to the adaptive rule-of-thumb in both charity and non-charity auction sets.

Step 3 For both charity and non-charity auctions, we compute the distribution and density of bids using the distribution and density of winning bids derived in Step 2 and the (n-1) order statistic.

Step 4 We first reveal the private values in both charity and non-charity auctions according bidding strategies. In particular, we reveal the private values in the charity auction set based on equation (9) with the estimated $\{\hat{\theta}, \hat{\lambda}\}$ from altruism parameters estimated (Section 7.1.2). For the private values in the non-charity auction set, we reveal them through $v_i = b_i$. We then nonparametrically estimate the distributions of revealed private values in both auction sets using the Gaussian kernel and bandwidth chosen according to the adaptive rule-of-thumb. These two estimated private value distributions are represented in Figure 4.

Step 5 We performed a Cucconi test to assess the equality of the two distributions of private values obtained from step 4.[17] The results of the test indicate thatthese two private

[17]The Cucconi test is a combination of location test and scale test, which is more powerful than a KS test because the KS test it is very sensitive to differences in both location and shape of the empirical cumulative distribution functions of the two samples.

value distributions are not statistically different ($p = 0.13$). ■

Moreover, to explore potential variation in altruistic parameters among different bidders, we conduct an over-identification test. This involves dividing the homogeneous winning bids from Step 1 in Section 7.1.1 into two subsamples based on their median. The altruistic parameters are subsequently estimated separately using these two subsamples. We expect that the two sets of altruistic parameters will not be statistically different, indicating the validity of assuming homogeneous altruistic parameters in the bidder population. The estimated results for θ and λ are (0.05, 0.00) for the winning bids below the median of all the winning bids, and (0.10, 0.00) for winning bids above the median. Based on a KS test, these two pairs of estimation are not statistically different ($p = 1$) suggesting that it is reasonable to assume the homogeneous altruistic parameters within our bidder population.

7.2 Comparison of Distributions of Private Values between Charity and Non-charity Auctions

7.2.1 Estimate Procedure

In this section, we compare the distributions of private values between charity and non-charity auctions. To obtain the private values for both charity and non-charity auctions, we utilize the estimated altruistic parameters obtained in the Section 7.1 and plug them back into equation (9). This allows us to reveal the private valuations on the auction items in charity auctions. Next, we calculate each bidder's total private value (v_i^j) in an individual charity auction by summing his private values on the auction item (v_i^p) and the linked-charity (v_i^p). Subsequently, we estimate the distribution of joint private values in charity auctions. Simultaneously, in the corresponding non-charity auctions, we employ the bidding strategy used in a second-price auction without charity, $v_i^p = b_i$, to reveal the private values. This allows us to estimate the corresponding distribution of private values in non-charity auctions. Finally, we compare these two distributions of private values using KS tests. The estimation steps are outlined as follows.

Step 1 To obtain the homogeneous winning bids, we first run the product-specific fixed effects model as in the equation (12) with donation dummy in vector **Donation**. We then take the $exp^{\epsilon_{im}}$ as the homogeneous winning bids from non-charity auctions and $exp^{\epsilon_{im}}$ plus the coefficient of **Donation** (β_1) as the homogeneous winning bids in the charity auction set.

Step 2 We nonparametrically estimate the distribution and density of winning bids with Gaussian kernels and bandwidths chosen according the adaptive rule-of-thumb in both charity and non-charity auction sets.

Step 3 Compute the distributions and densities of all the bids in both charity and non-charity auctions using the distributions and densities obtained from Step 2 and the (n-1) order statistic.

Step 4 To reveal the private values in charity auction set, we use equation (9) with the estimated $\{\hat{\theta}, \hat{\lambda}\}$ obtained from the altruistic parameters estimation process. Meanwhile, in the non-charity auction set, the private values are revealed as $v_i^p = b_i$. Afterwards, we estimate the distributions of private values in both auction sets using the revealed private values. In the charity auction set, the distribution represents the distribution of the joint private values, considering both the private values on the auction item and the linked-charity. On the other hand, in the non-charity auction set, the distribution corresponds to the evaluations solely on the auction item.

Step 5 We conduct a KS test to assess whether these two distributions of private values obtained from step 4 are statistically the same. Additionally, we perform a one-sided KS test to determine whether the distribution of joint value lies blow the distribution of auction item value using a one-sided KS test.

7.2.2 Estimation Result

The estimated result is report in Figure 3. The solid red line represents the distribution of joint private values from the charity auction set, while the dashed blue line represents the distribution of private values from the non-charity auction set. Notably, the average price in charity auction set is statistically higher than it in the non-charity auction set, as evidenced by the domination of non-charity set's distribution. This finding aligns with the finding in (Elfenbein and McManus, 2010) that bidders bid more aggressive in charity auctions.

Moreover, we find that the two distributions of private values from charity and non-charity auction sets are statistically different, as indicated by both the Cucconi and KS tests. Additionally, we conduct an one-sided KS test with the null hypothesis that the distribution of joint value not greater than the distribution of value of auction item, and the alternative hypothesis that the distribution of joint value lies above that of value of auction item. The test statistic, D^+, is 3.45 with $p = 1$. Thus, we cannot reject the null hypothesis. This suggests that the joint value is stochastically bigger than the value of auction item. In other words, the distribution of joint value lies below the distribution of value of auction item, implying the difference between these two distributions or the charity/public goods valuations linked to charity auctions are statistically significant in our data.

7.3 The distribution of public goods values

In this section, we investigate the distribution of public goods values using the complete combinatorial method introduced in the Section 3.3B. Specifically, within each pair of charity and non-charity auctions, we determine the public good value by taking the difference between the private values obtained from the charity and non-charity auctions. Although we have revealed the two sets of private values in Section 7.2, there is still one more issue to address before we apply the complete combinatorial method here. Recall that, on average, each charity auction is matched with 2.4 non-charity auctions. However, for the complete combinatorial method to be applicable, it requires that the two samples have the same length. To address this discrepancy, within each pair of charity and non-charity auctions, we calculate the average private values across the non-charity auctions and consider it as the private value in the corresponding non-charity auction. Figure 6 shows the estimated distribution of public goods values obtained from this method.

8 Conclusion

In this paper, we investigated second-price auctions with and without charity. Using the independent private values paradigm, we were able to identify the underlying distribution of bidders' private values from the observed winning bids without making any parametric assumptions. By utilizing the matched data structure of identical charity and non-charity auctions, we proposed a nonparametric approach to estimate the marginal utility of charity and the distribution of bidders' private values linked to the auction item.

Applying this approach to publicly available eBay data from Elfenbein and McManus (2010), our findings reveal positive values for both warm glow and pure altruism. The results suggest that individuals are willing to pay more for cause-linked products primarily due to warm glow, although pure altruism is positive. Specifically, the magnitude of warm glow is 9 cents from each one dollar donation. The magnitude of pure altruism is positive but pretty close to zero. Additionally, we estimate the distribution of the value of charity revenue as a public good by taking the difference between the distributions of private values in the charity and non-charity auction sets.

The methodology presented in this paper has several important implications. For governments, it provides a way to identify the underlying altruistic parameters, which could inform the design of tax and subsidy policies. This approach enables more accurate estimation of the impact of taxes on charitable giving, potentially leading to more effective tax policies. As Andreoni (1990, p. 471) has noticed, the relative degrees of altruism are of primary importat in determining tax and subsidy policies. He points out that when we estimate how tax changes will impact charitable giving, accounting for the degrees of altruism may potentially sometimes yield conclusions dramatically different from those drawn without it.

Furthermore, this approach has potential implication for social welfare improvement, as the correlation between pure and impure altruism has implications for the charity to which the collected money from charity auctions will flow. When the impure altruism appears, especially when it dominates among motivations, givers care more about the act of giving itself rather than the linked charity, resulting in a mismatch between the charity receiving funds and the most valuable cause in need. Given the substantial donor base in cause-related markets, even a small magnitude of altruistic motivation can lead to a significant amount of giving. Consequently, if governments could guide this large amount of giving to the most valuable cause, the improvement in social welfare would be remarkable.

Additionally, our approach has potential implications for the design of fundraising campaigns design and the design of firms' optimal social responsibility strategies. Understanding whether donors are primarily motivated by altruism, warm glow, or a combination of both, as highlighted by Vesterlund (2017), is crucial for effective fundraising appeals, interventions, and public policies. Our method quantifies the magnitudes of altruistic and warm glow motivations, providing valuable insights for designing more effective fundraising appeals. Moreover, the structural approach we employ enables the development of optimal corporate social responsibility strategies. This is achieved through the ability of our approach to conduct simulations with varying donation percentages, enabling a thorough exploration of different scenarios and assisting in the identification of optimal strategies. While our analysis focuses on a single cause-related market, specifically Giving Works on eBay, our approach can be extended to other causes and auction formats, which we plan to explore in future research.

In conclusion, this paper contributes to the structural analysis of charity auction data by proposing a convenient way to estimate the marginal utility and the distribution of public good in a cause-related market. Our nonparametric approach serves as a crucial link between a behavioral model and field data, allowing for the structural estimation of the underlying parameters. This integration of theory and empirical analysis provides a robust foundation for understanding the auctions with and without charity. By bridging the gap between theoretical frameworks and real-world observations, our approach contributes to the advancement of research in this field and lays the groundwork for further investigations in auction theory and charitable giving.

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Tables

	Obs.	Mean	Median	Min.	25%	75%	Max.
Panel A Charity Auctions							
Price (including shipping)	723	102.38	50.59	8.11	28.54	111.03	1371.01
Length of auction (days)	723	6.51	7.00	1.00	7.00	7.00	10.00
Number of bids	654	9.99	9.00	1.00	5.00	13.00	57.00
Number of bidders	654	5.21	5.00	1.00	3.00	7.00	21.00
Seller rating	723	3.50	275	1.00	71.00	4061	139266
Donation percentage	723	66.15	100.0	10.00	10.00	100.00	100.00
Panel B Non-charity Auctions							
Price (including shipping)	1710	95.78	48.00	5.89	26.46	101.48	1405.00
Length of auction (days)	1710	5.29	7.00	1.00	3.00	7.00	10.00
Number of bids	1257	10.38	9.00	1.00	4.00	15.00	54.00
Number of bidders	1257	5.76	5.00	1.00	3.00	8.00	20.00
Seller rating	1710	6677.50	400.5	1.00	86.00	1992.5	309979

Table 1: Summary Statistics of Auctions

Initial values	Estimated values
0.5, 0.1	0.478, 0.083
0.75, 0.25	0.729, 0.221
0.6, 0.3	0.622, 0.316
0.9, 0.1	0.862, 0.074
0.8, 0.4	0.82, 0.42
1, 0.4	0.96, 0.37

Sample size is 1000 auctions, the distribution of value is uniform distribution on [0,1], and the number of repeat is 100.

Table 2: Simulation with different pairs of initial values

Figures

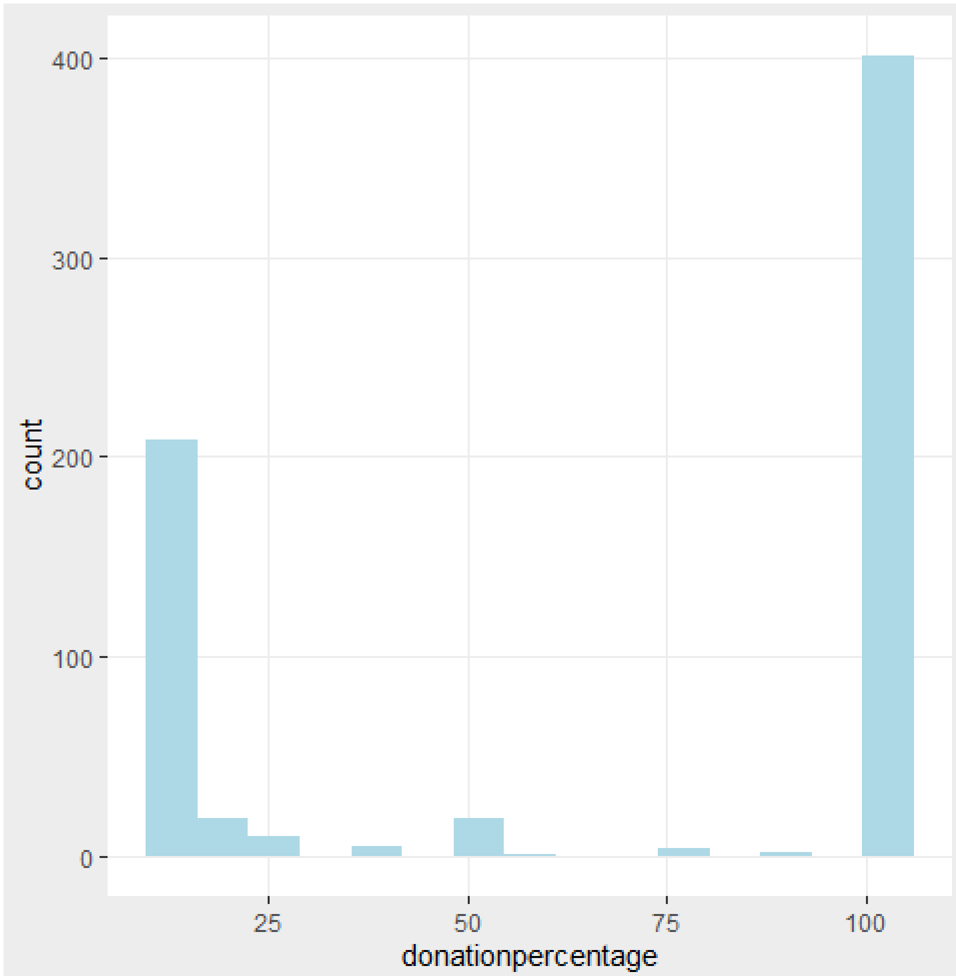

Figure 1: The histogram of donation percentage

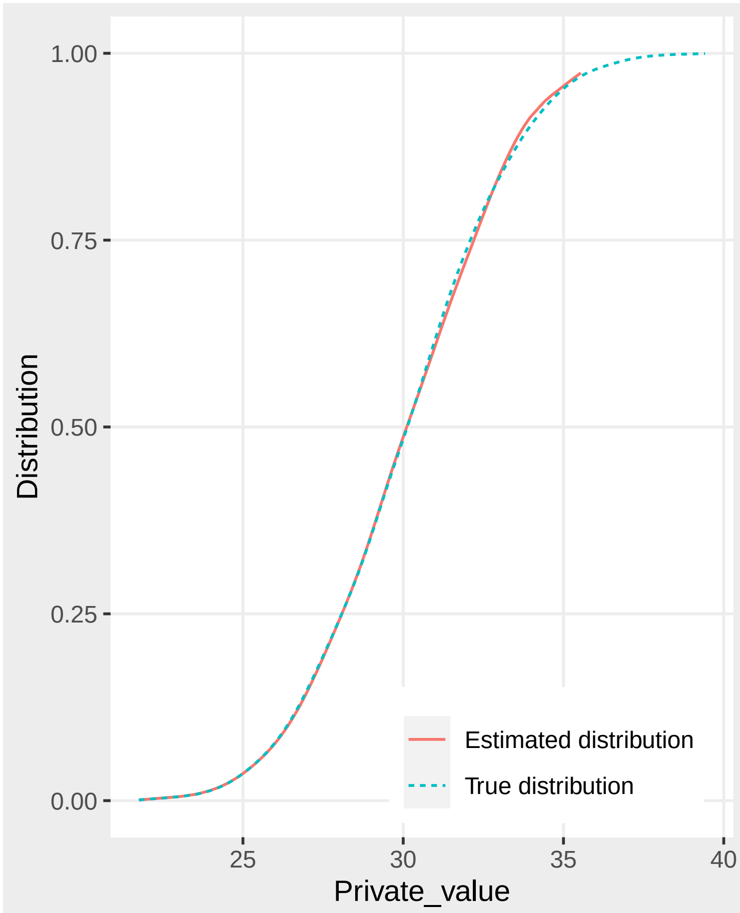

Figure 2: The comparison between the true and the estimated distribution of bids non-charity auction set

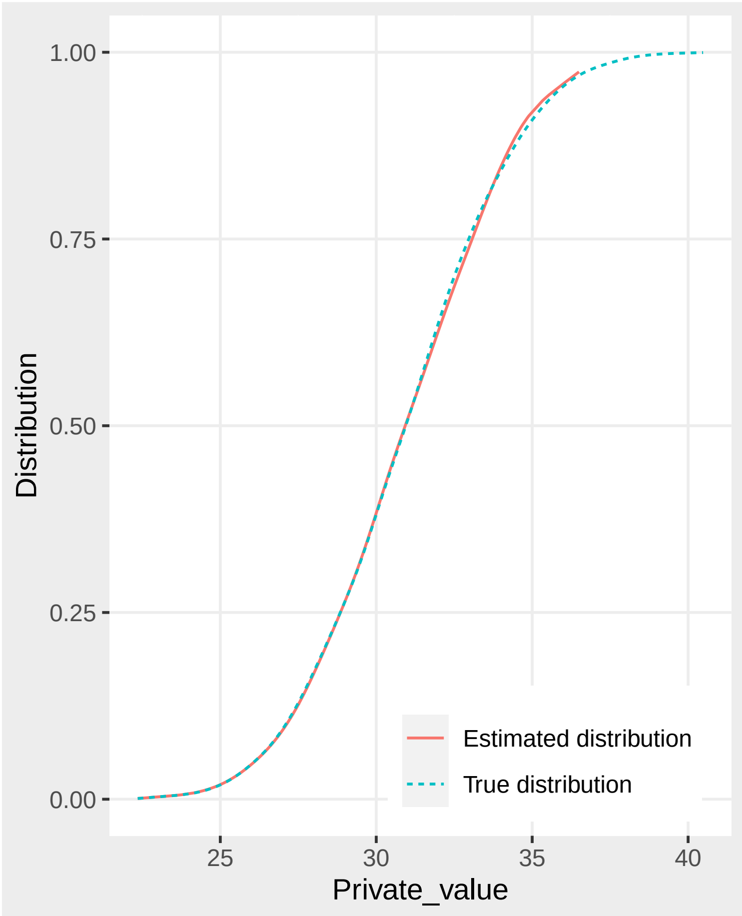

Figure 3: The comparison between the true and the estimated distribution of bids in charity auction set

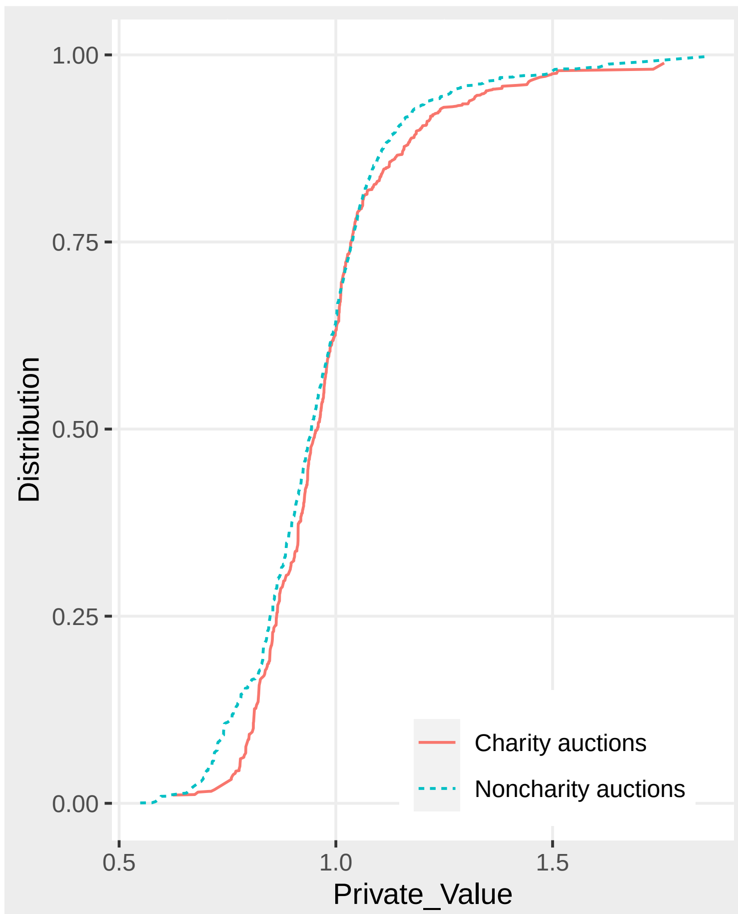

Figure 4: The comparison between the between the distributions of private values on auction items in charity and noncharity auction sets

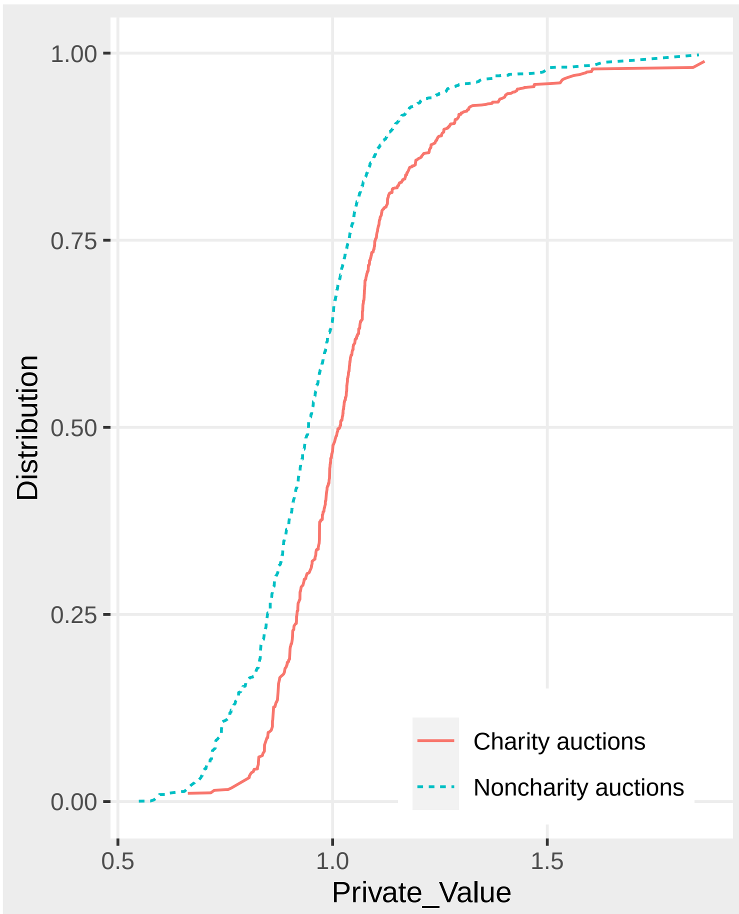

Figure 5: The comparison of private value distribution from charity and non-charity auctions

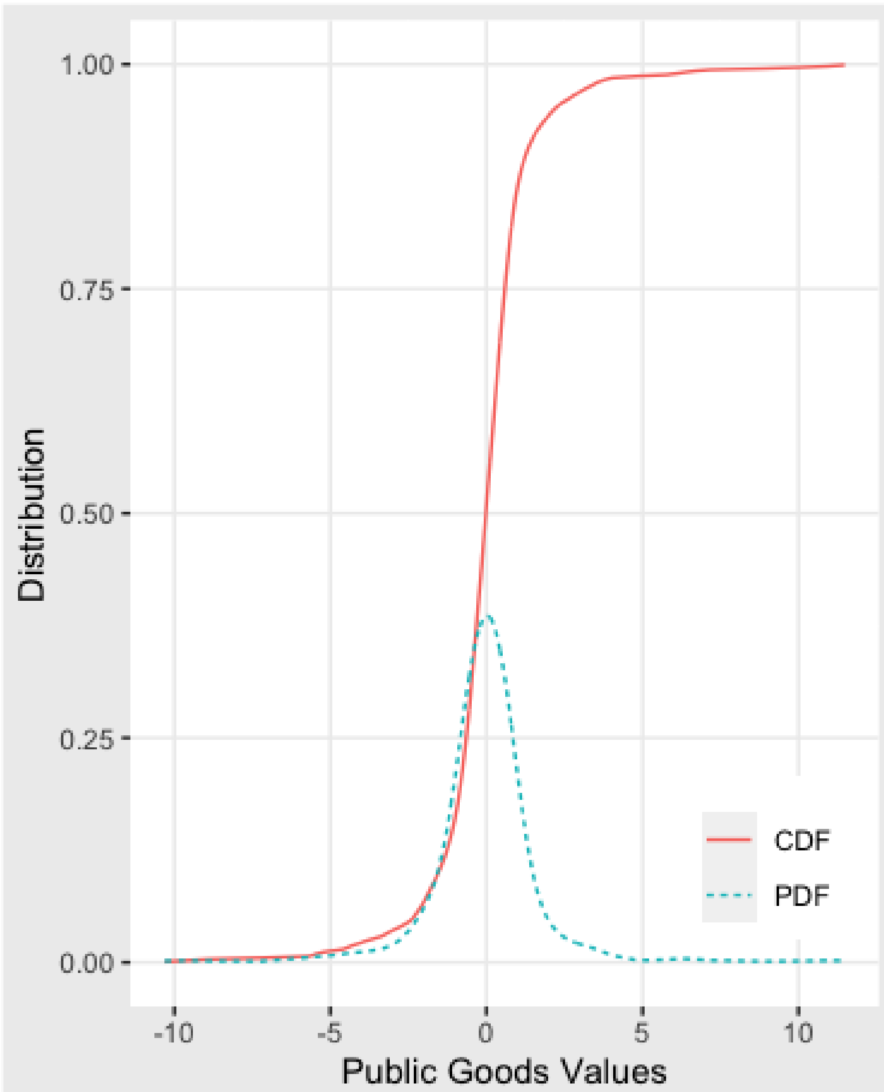

Figure 6: The Distribution of public values

Appendix A.Charity Dummies

Education Education for charities that work and serve students from every age group, preschool to graduate school and beyond. It includes charities serving as educational institutions and charities focusing on making education more accessible and effective for different people and communities.

Healthcare Healthcare for charities dealing with healthcare and health research.

Environmental Charities This dummy for charities that focus on preservation, appreciation, and sustainable development for the environment. There are two primary subgroups:

- Environmental conservation and protection
- Parks and nature centers

Religion It for charities that serve religious activities and religious media and broadcasting.

Human Services It for charities that focus on activities in following groups:

- Children's and Family Services
- Youth Development, Shelter, and Crisis Services
- Food Banks, Food Pantries, and Food Distribution
- Multipurpose Human Service Organizations
- Homeless Services
- Social Services

International Charities These are typically charities that have headquarters in one country but do a lot of work in numerous other locations and countries.

Animal Charities Animal charities are just what they sound like—a way to support, protect, and conserve animals and wildlife.

Human and Civil Rights It for charities that take action to defend civil rights, such as protestors' rights, immigrants' rights, voting rights and so on.

Research and Public Policy It for non-medical science and technology research, and social and public policy research.

Arts, Culture, Humanities It includes three subgroups:

- Libraries, Historical Societies and Landmark Preservation
- Museums
- Sports

Community Development They function to support and revitalize communities, especially those that are impoverished or struggling. Affordable housing projects are the most common example of these charities.

Foreign Charities It deals with issues outside of the US.

Appendix B. The derivative of the bid function

$$\begin{aligned} \pi(b_i \mid v_i, n) = & \int_{\underline{v}}^{\phi(b_i)} [v - B(x) + \delta\theta B(x)] dF(x)^{n-1} \\ & +\delta\lambda(n-1)F(\phi(b_i))^{(n-2)}[1 - F(\phi(b_i))]B(b_i) \\ +\delta\lambda \int_{\phi(b_i)}^{\bar{v}} & B(x)(n-1)(n-2)F(x)^{n-3}[1-F(x)]dF(x) \\ = & \int_{\underline{v}}^{\phi(b_i)} [v - B(x)] dF(x^{n-1}) + \Phi(b_i) \end{aligned} \tag{14}$$

with

$$\Phi(b_i) = \delta\theta \int_{\underline{v}}^{\phi(b_i)} B(x) dF(x^{n-1}) + \delta\lambda(n-1)F(\phi(b_i))^{n-2}[1 - F(\phi(b_i)]B(b_i)$$

$$+\delta\lambda \int_{\phi(b_i)}^{\bar{v}} B(x)(n-1)(n-2)F(x)^{n-3}[1-F(x)]dF(x)$$

The term Φ is increasing in b_i, suggesting that participants in a charity auction bid more aggressively than in a standard auction:

$$\Phi'(b_i) = \delta\lambda(n-1)F(\phi(b_i))^{n-2}[1 - F\phi(b_i))]B'(b_i) + \delta(\theta - \lambda)B(b_i)\frac{dF(\phi(b_i))^{n-1}}{db_i}$$

Applying the first-order condition for an incentive-compatible selection of b_i,

$$\frac{d\pi(b_i \mid v, n)}{db_i} |_{b_i=v_i}= 0$$

we obtain the first-order condition:

$$[v_i - (1 - \delta(\theta - \lambda)B(v_i)]\frac{dF(v_i)^{n-1}}{dv} + \delta\lambda(n-1)F(v_i)^{n-2}[1 - F(v_i)]B'(v_i) = 0$$

Simplifying it yields the differential equation:

$$B'(v_i) - B(v_i)(\frac{1 - \delta(\theta - \lambda)}{\delta\lambda})(\frac{f(v_i)}{1 - F(v_i)}) = -\frac{v_i}{\delta\lambda}(\frac{f(v_i)}{1 - F(v_i)})$$

Let β denotes $\frac{1-\delta(\theta-\lambda)}{\delta\lambda}$ and multiply each side of it by the integrating factor $[1 - F(v_i)]^{\beta}$ to obtain

$$\frac{d}{dv_i}\{B(v_i)[1 - F(v_i)]^{\beta}\} = (\frac{v_i}{1 - \delta(\theta - \lambda)})\frac{d}{dv_i}\{[1 - F(t)]^{\beta}\}$$

Then we integrate both sides and use the boundary condition, we get our bid function for a second-price charity auction:

$$B(v_i) = (\frac{1}{1 - \delta(\theta - \lambda)})\{v_i + \int_{v_i}^{\bar{v}} (\frac{1 - F(x)}{1 - F(v_i)})^{\beta} dx\} \tag{15}$$

Appendix C. Monte Carlo Simulation with Normal Distribution

In this section, we repeat the Monte Carlo simulation in Section 5 with an normal distribution on (30,3). In both charity and non-charity auction sets, the true and estimated distributions of private values are almost overlapped, see Figure 7 and Figure 8. We also run a KS test ($p = 0.00$) implying the estimated distributions of private values are statistically the same as the true underlying distribution of private values in both charity and non-charity auction sets. This result of equality-test combined with the comparison between estimated and true private value distributions supports that this approach can achieve consistent estimate of the underlying private value distribution using the winning bids only regardless the format of underlying distribution of private values.

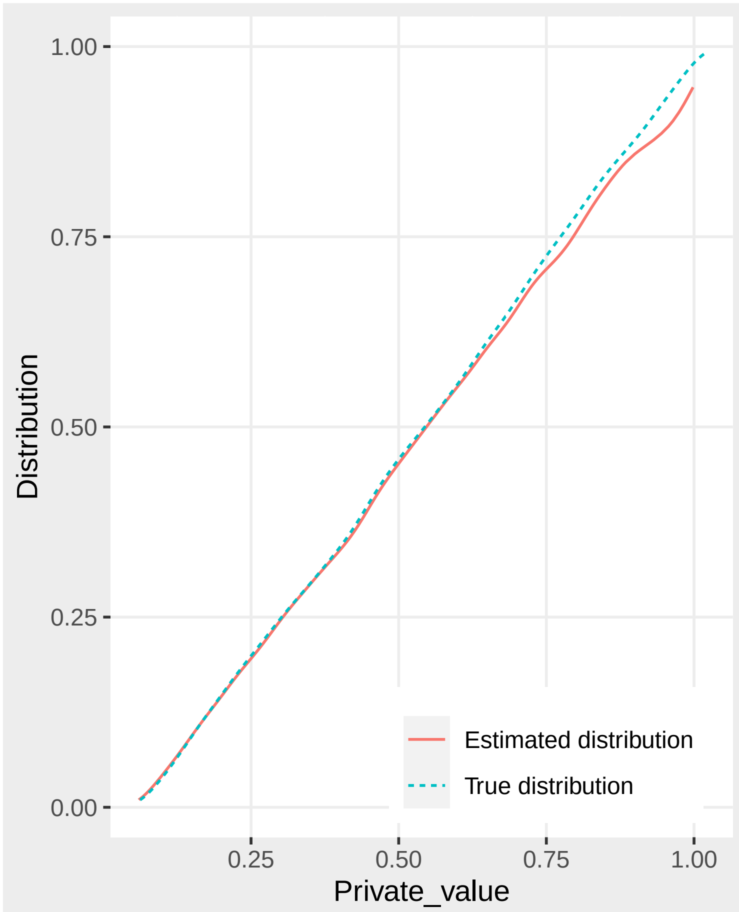

Figure 7: The comparison between the true and estimated private value distributions in charity auction set

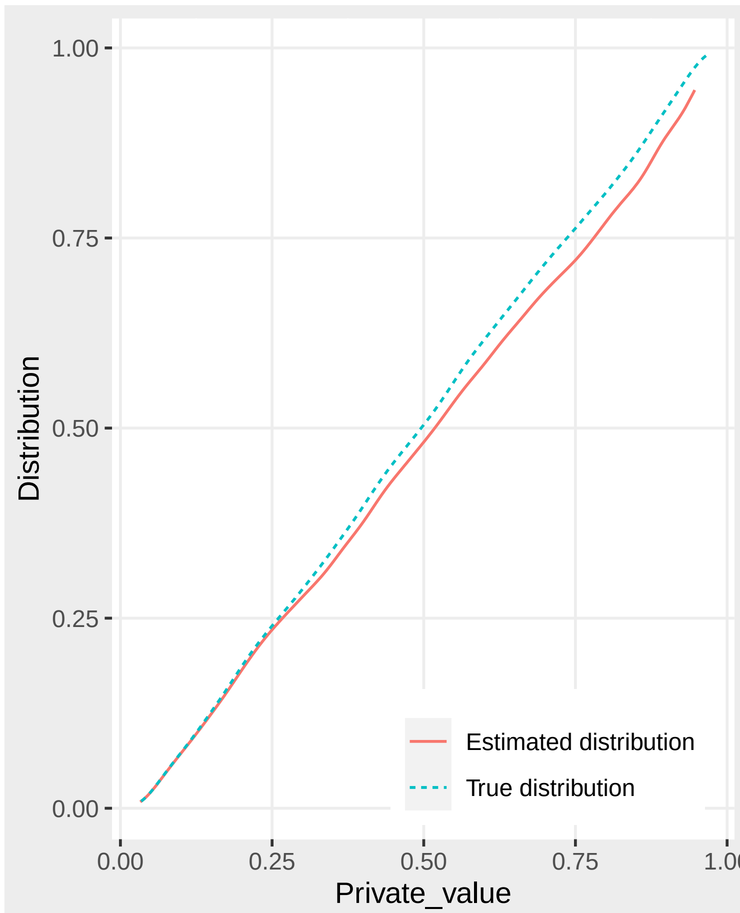

Figure 8: The comparison between the true and estimated private value distributions in non-charity auction sets

Appendix D. Replicated results

Dependent variable	log(price+shipping)	log(price+shipping)	log(price)
CHARITY	0.061(0.008)	0.063(0.008)	0.07(0.08)
10%- SHARE	0.054(0.014)	0.054(0.014)	
100%-SHARE	0.073(0.010)	0.074(0.010)	
MID-SHARE	0.028(0.022)	0.029(0.021)	
Length	0.003(0.002)	0.004(0.002)	
log(seller rating)	0.004(0.002)	0.003(0.002)	
Power seller dummy	0.018(0.011)	0.017(0.011)	
Buy-it-now dummy	0.048(0.010)	0.052(0.010)7	
The number of bidders included?	No	Yes	No
Bidders' feedback	No	No	Yes

Notes: For each model, there are 2433 observations and 723 groups. The models contain a fixed effect for each match/product. The standard errors are clustered at the group level, which is in the parentheses.

Table 3: Charity premium estimates